Numerical Analysis on Run-Up of Multi-Solitary Waves on A Planar Slope

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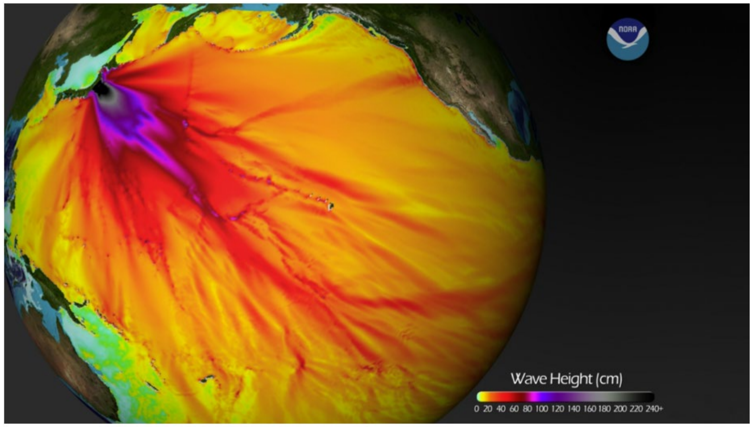
Tsunami waves

- A series of extremely long waves

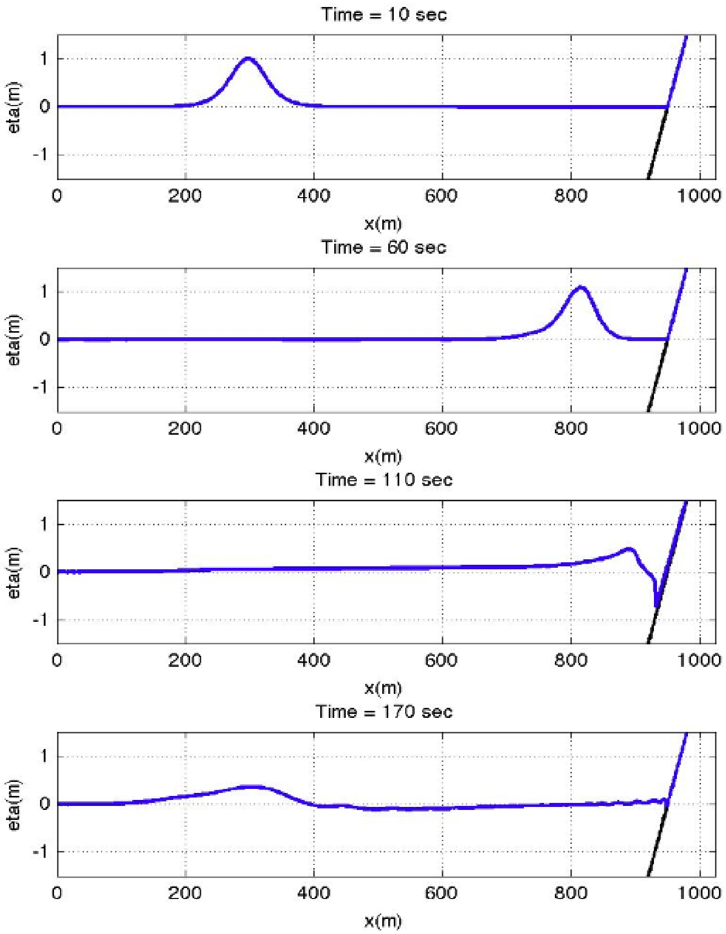
- Caused by a large and sudden displacement of the ocean (earthquakes)

- Transform into a series of **solitary waves** or undular bores over a mild slope



Expected tsunami wave heights from the March 2011 Honshu, Japan undersea earthquake. (Image credit: NOAA Center for Tsunami Research)

FUNWAVE-TVD



- Solves fully nonlinear Boussinesq equations
- A high-order adaptive time-stepping numerical scheme
- A total variation diminishing scheme
- “Waves on a 1D slope” demo

fyshi, Malej, M., fengyanshi, mayhl, Ye, Z., JimKirby, & Brandt, S. R. (2021). fengyanshi/FUNWAVE-TVD: Version_3.6. doi:10.5281/ZENODO.5039418

Governing Equations

The FUNWAVE model addresses the fully nonlinear Boussinesq equations (Shi et al., 2012):

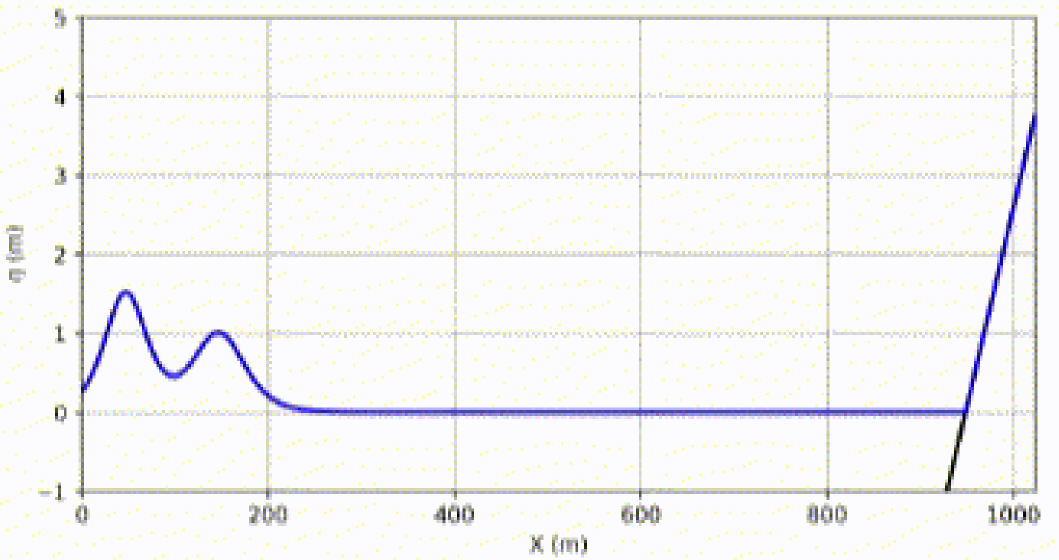
$$\zeta_t + \nabla \cdot \mathbf{M} = 0 \tag{1}$$

$$\begin{aligned} \mathbf{M} = (h+\zeta)\Bigg[\mathbf{u}_\alpha + \left(\frac{z_\alpha^2}{2} - \frac{1}{6}\left(h^2 - h\zeta + \zeta^2\right)\right)\nabla\left(\nabla \cdot \mathbf{u}_\alpha\right) \\ + \left(z_\alpha + \frac{1}{h}(h-\eta)\right)\nabla\left(\nabla \cdot (h\mathbf{u}_\alpha)\right)\Bigg] \end{aligned} \tag{2}$$

$$\mathbf{u}_{\alpha t} + \left(\mathbf{u}_\alpha \cdot \nabla\right)\mathbf{u}_\alpha + g\nabla\zeta + \mathbf{V}_1 + \mathbf{V}_2 = 0 \tag{3}$$

where $\eta = h + \zeta$ is the total water depth, ζ is the free surface elevation, g is the downwards gravitational acceleration, $\mathbf{u}_\alpha$ is the reference velocity and $\mathbf{V}_1$ and $\mathbf{V}_2$ are the dispersive terms defined in Kennedy et al. (2000). where $u(x,t)$ is the horizontal velocity, η is the total water depth, and $\zeta = \eta - h$ is the free surface elevation.

Run-Up of 1D Solitary Waves (Modified)



Experiments to be Compared with

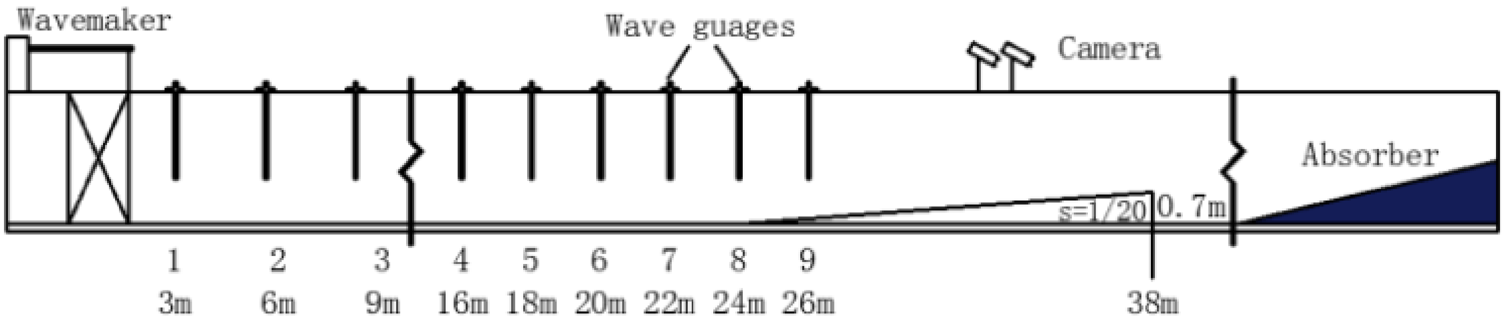


Fig.1 Sketch of the experimental setup.

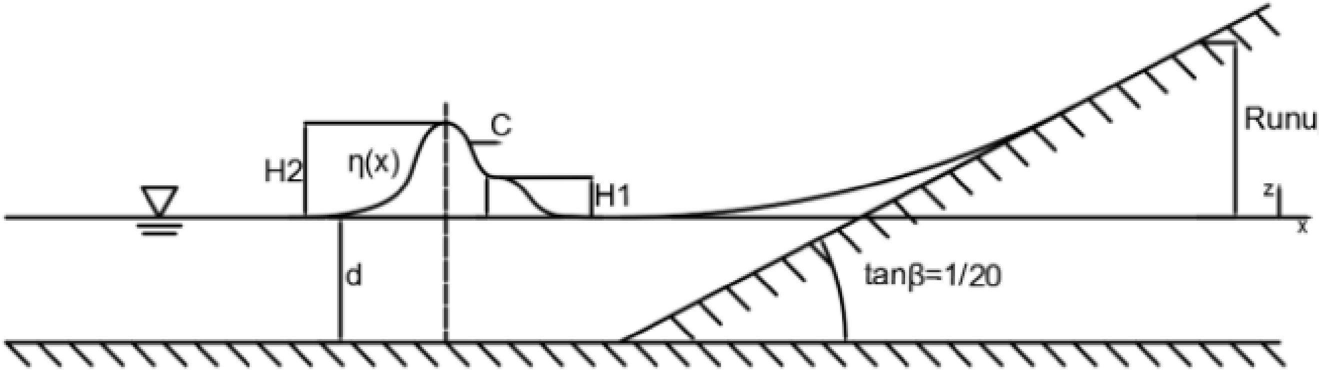
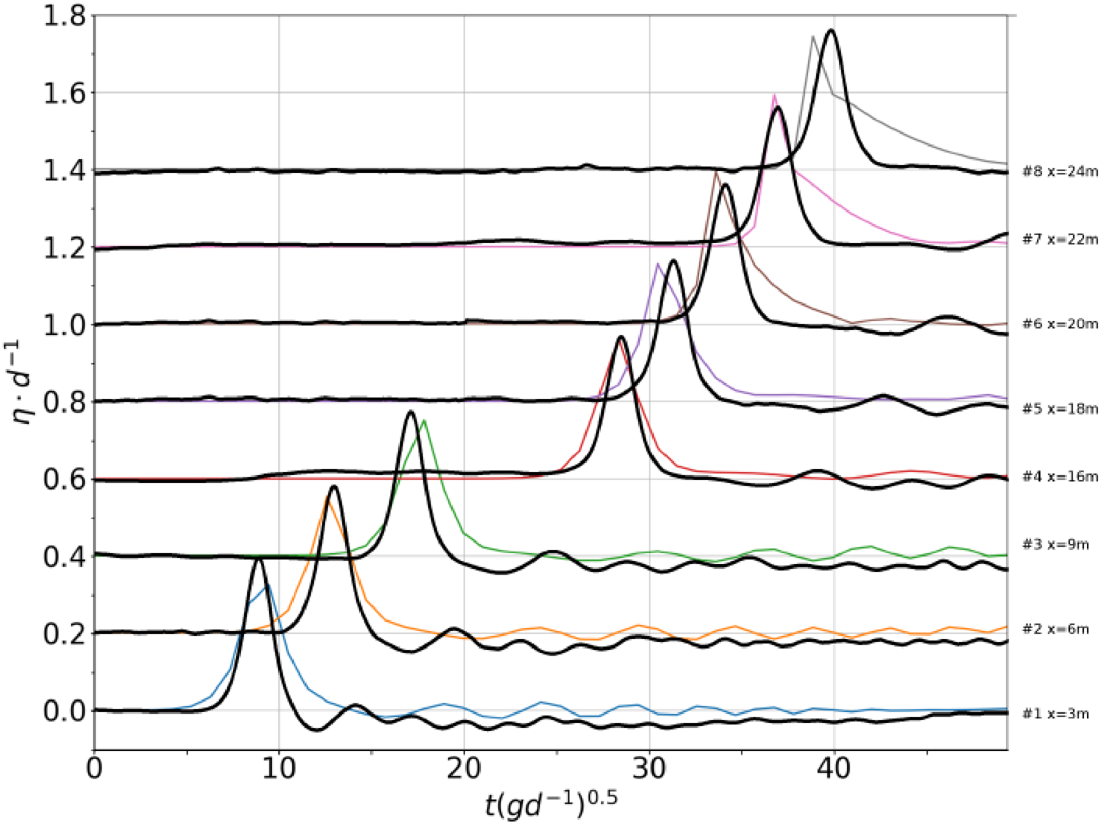


Fig.2 Sketch of runup of double solitary waves on a slope.

(Liu et al., 2018)

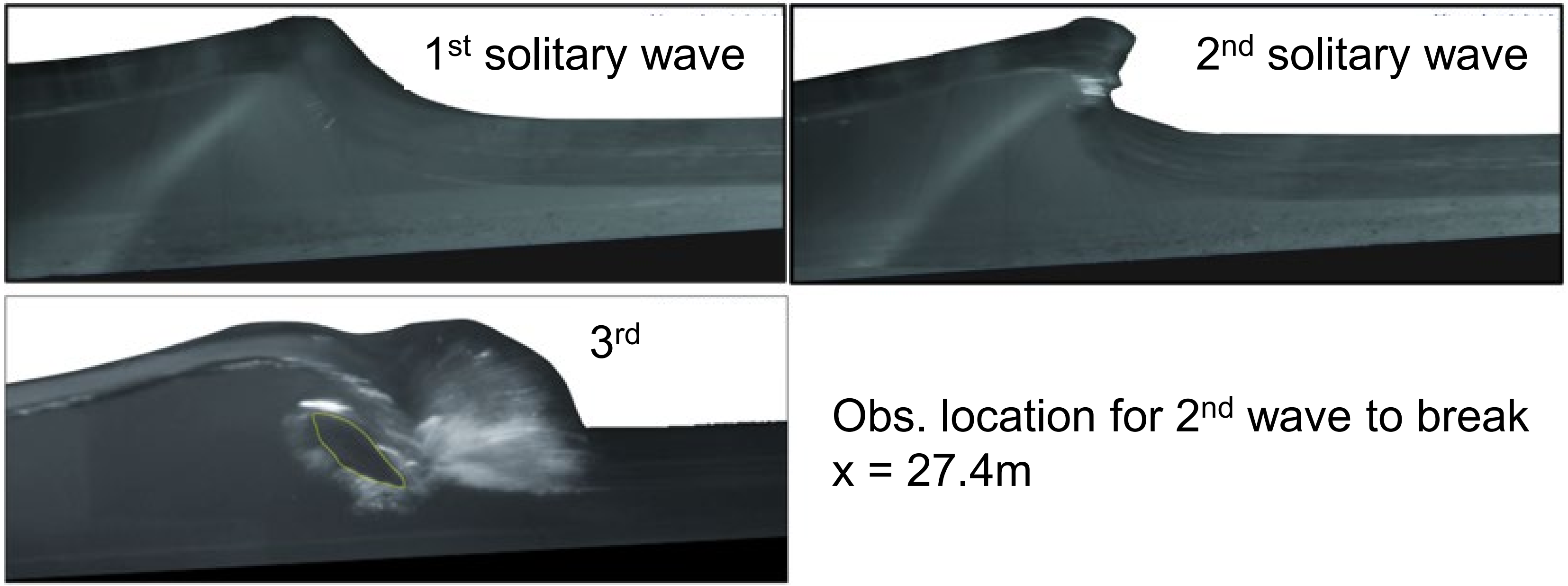
Model Validation: Single Solitary Wave Run-Up



(b) Time series of free surface elevation with $d = 0.4$m and $H/d = 0.361$

Fig. 1 Snapshots and time series of single-solitary wave

Triple Solitary Waves (Experiment)



Obs. location for 2nd wave to break
x = 27.4m

Fig.3-7 Wave breaking position of triple solitary waves on slope.（d=0.4m,H/d=0.361, ε=0.8）

(Peng, 2018)

Triple Solitary Waves (Numerical)

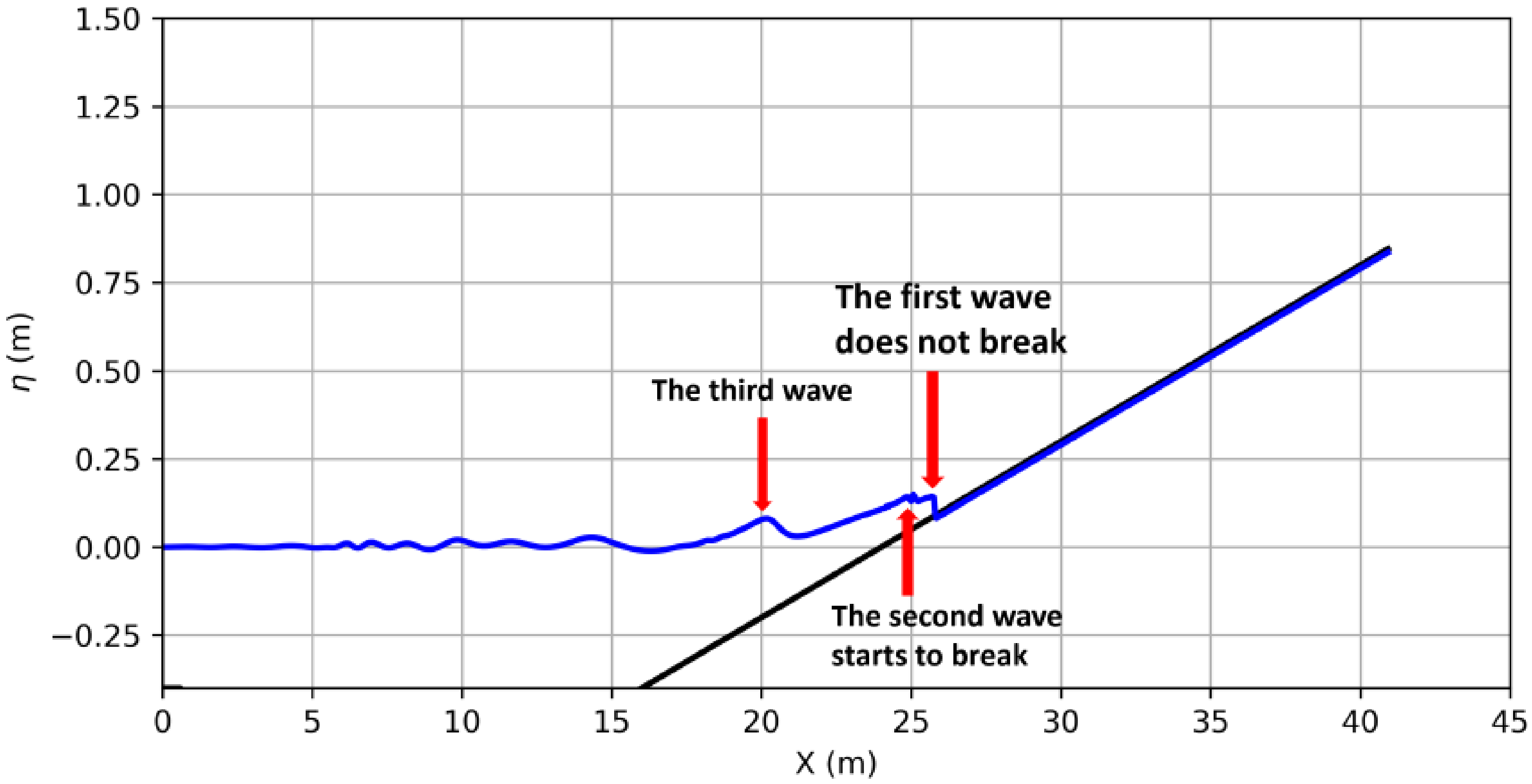


Fig. 2 Simulated breaking location for the second solitary wave

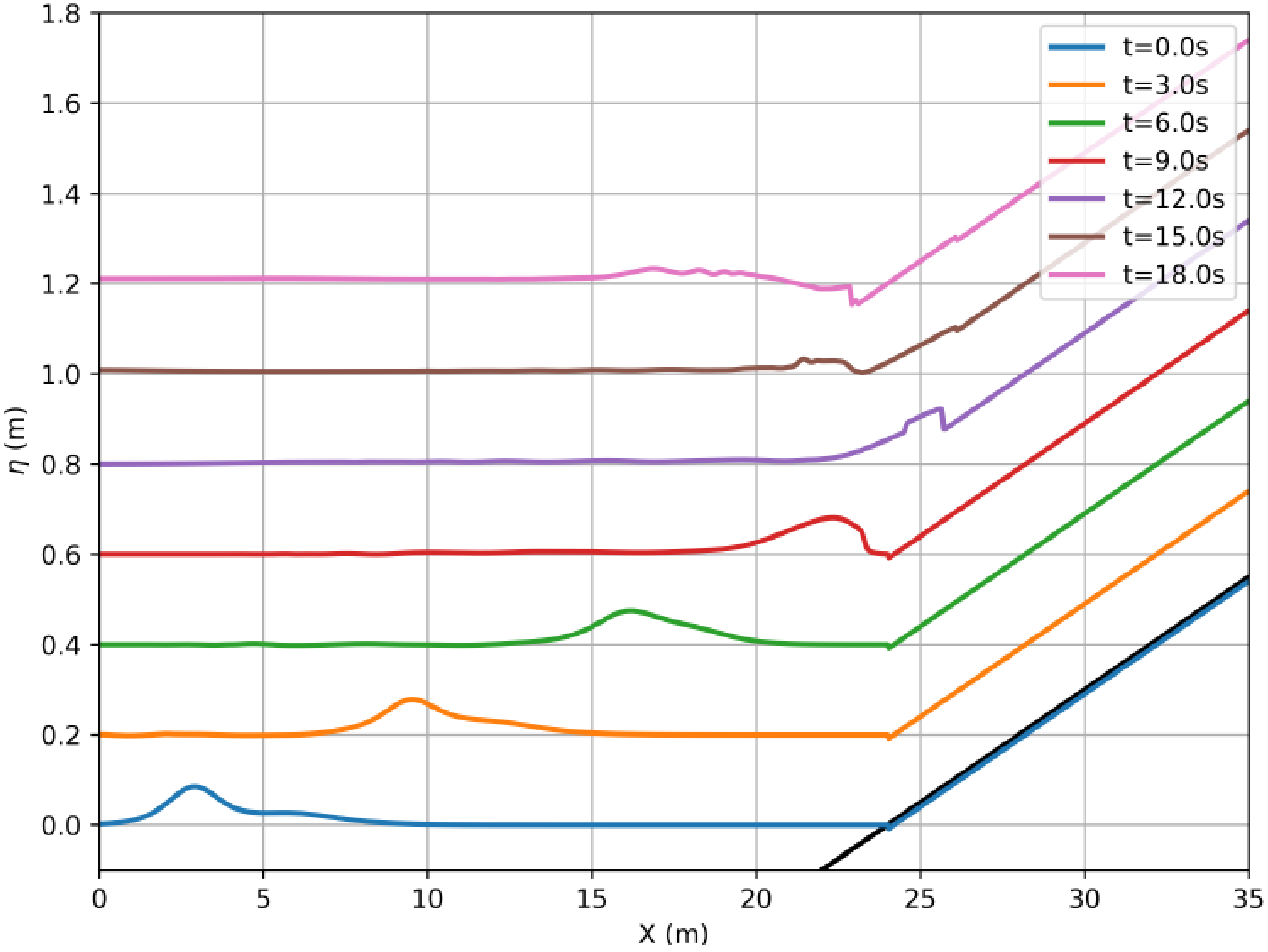
Overtaking of Double-Solitary Waves

Table1: Conditions for double-solitary waves, $d = 0.3$m.

Case	H_1 (m)	H_2 (m)	ϵ	H_2/H_1
I	0.0465	0.0816	0.4	1.755
II	0.0465	0.0816	1.0	1.755
III	0.0252	0.0816	0.4	3.238
IV	0.0252	0.1113	0.4	4.417

$$X_2 = X_1 - \frac{\epsilon\sigma_1}{k_2} = X_1 - \epsilon\sqrt{\frac{H_1}{H_2}}\left(\frac{2\pi}{k_1} - \frac{2H_1}{k_1 d}\right) = X_1 - 2\epsilon\sqrt{\frac{4d^3}{3H_1}}\left(\pi - \frac{H_1}{d}\right)$$

Overtaking Case III



Overtaking Case III

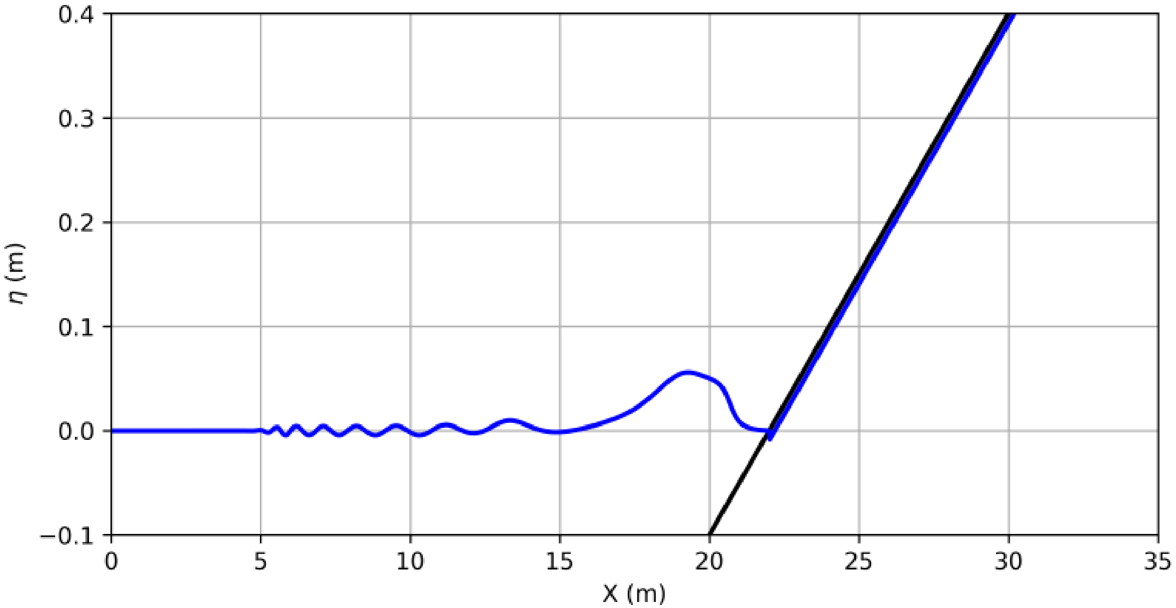
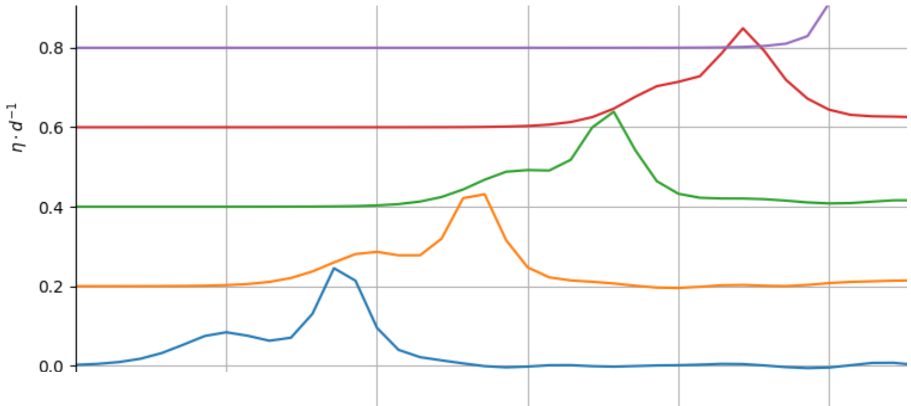
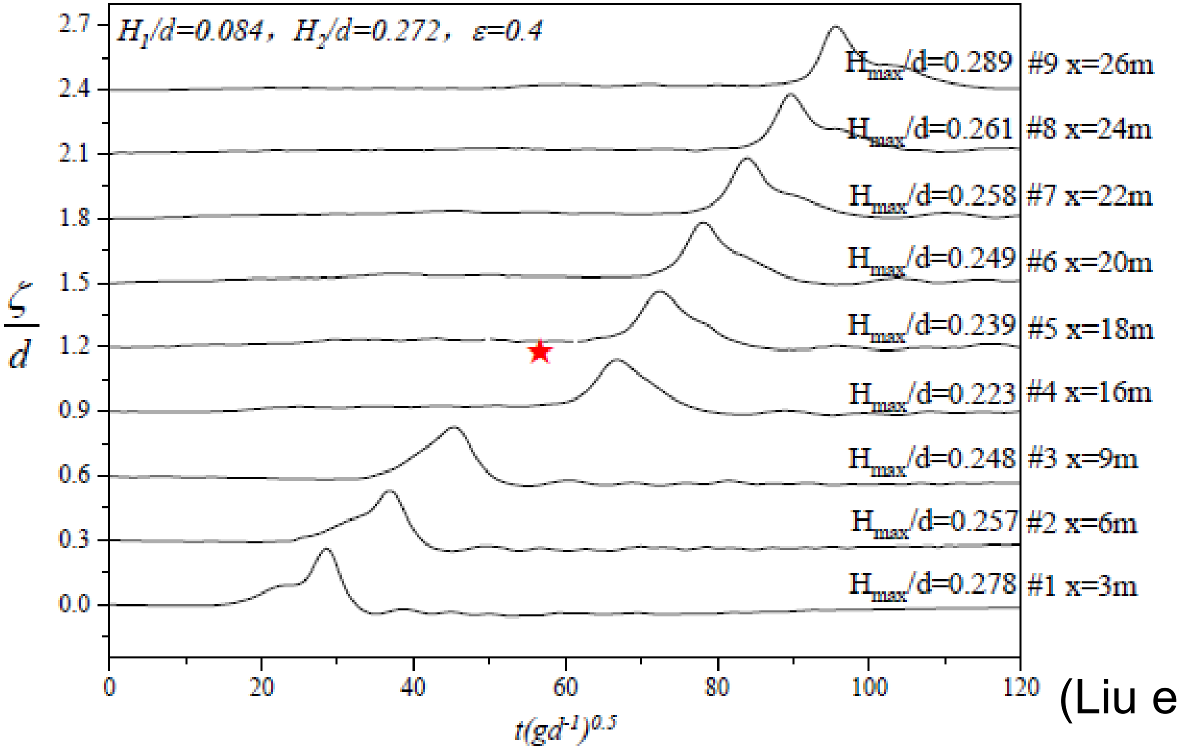


Fig. 4 Snapshot of Case III flat peak at t = 9.75s

Recapture of overtaking location





(Liu et al., 2018)

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Takeaways

- The modified FUNWAVE-TVD model is an effective tool in capturing the runup and overtaking of **multi-solitary waves** in the one-dimensional sense.

- Key qualitative results including the **wave-breaking location** for triple-solitary waves, and the **overtaking location** for double-solitary waves have been reproduced.

- Future works can focus on the **convergent runup height** of a large number of solitary waves.

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